Evaluate $\int \frac{dt}{1-e^t}$.

SCORE: _____ / 6 PTS

SOLUTION 1:

(1) $u = 1 - e^t \quad \Rightarrow \quad t = \ln(1-u)$

$dt = \frac{1}{u-1}\,du$

(1/2) $\int \frac{1}{u(u-1)}\,du = \int (\frac{A}{u} + \frac{B}{u-1})du$

(1/2) $1 = A(u-1) + Bu$

$u = 1: \quad 1 = B$

$u = 0: \quad 1 = -A \quad \Rightarrow \quad A = -1$

SANITY CHECK: $u = 2: \quad \frac{1}{2(1)} \overset{?}{=} \frac{-1}{2} + \frac{1}{1} = \frac{-1+2}{2} = \frac{1}{2}$ ✓ (1/2)

(1) (1) $\int (-\frac{1}{u} + \frac{1}{u-1})du = -\ln|u| + \ln|u-1| + C$ (1/2)

$= -\ln|1-e^t| + \ln|-e^t| + C$

$= t - \ln|1-e^t| + C$ (1)

OR

SOLUTION 2:

$\int \frac{1}{1-e^t}\frac{e^{-t}}{e^{-t}}\,dt = \int \frac{e^{-t}}{e^{-t}-1}\,dt$ (2)

$u = e^{-t} - 1$ (1)

$du = -e^{-t}\,dt$

(1) $\int -\frac{1}{u}\,du = -\ln|u| + C$ (1)

$= -\ln|e^{-t} - 1| + C$

(1)

Evaluate $\int \ln(t^2 - t + 2)\, dt$.

SCORE: _____ / 8 PTS

$u = \ln(t^2 - t + 2) \qquad dv = dt$

$du = \frac{2t-1}{t^2-t+2}\, dt \qquad v = t$

$t\ln(t^2 - t + 2) - \int \frac{2t^2 - t}{t^2 - t + 2}\, dt$

$= t\ln(t^2 - t + 2) - \int (2 + \frac{t-4}{t^2-t+2})dt$

$= t\ln(t^2 - t + 2) - \int (2 + \frac{\frac{1}{2}(2t-1) - \frac{7}{2}}{t^2-t+2})dt$

$= t\ln(t^2 - t + 2) - 2t - \frac{1}{2}\ln(t^2 - t + 2) + \frac{7}{2}\int \frac{1}{(t-\frac{1}{2})^2 + \frac{7}{4}}\, dt$

$= t\ln(t^2 - t + 2) - 2t - \frac{1}{2}\ln(t^2 - t + 2) + \frac{7}{2}\frac{2}{\sqrt{7}}\tan^{-1}\frac{t-\frac{1}{2}}{\frac{\sqrt{7}}{2}} + C$

$= t\ln(t^2 - t + 2) - 2t - \frac{1}{2}\ln(t^2 - t + 2) + \sqrt{7}\tan^{-1}\frac{2t-1}{\sqrt{7}} + C$

(1) EACH

Evaluate $\int (1-\sqrt{t})^{20}\, dt$.

SCORE: _____ / 5 PTS

SOLUTION 1:

(1) $u = 1-\sqrt{t} \quad \Rightarrow \quad t = (1-u)^2$

$dt = -2(1-u)du$

(1) $\int -2(1-u)u^{20}du = \int (-2u^{20} + 2u^{21})du$ (1)

(1) $= -\frac{2}{21}u^{21} + \frac{1}{11}u^{22} + C$

$= -\frac{2}{21}(1-\sqrt{t})^{21} + \frac{1}{11}(1-\sqrt{t})^{22} + C$ (1)

OR

SOLUTION 2:

(1) $u = (1-\sqrt{t})^{20} \quad \Rightarrow \quad t = (1-u^{\frac{1}{20}})^2$

$dt = -\frac{1}{10}u^{-\frac{19}{20}}(1-u^{\frac{1}{20}})du$

(1) $\int -\frac{1}{10}u^{\frac{1}{20}}(1-u^{\frac{1}{20}})du = \int (-\frac{1}{10}u^{\frac{1}{20}} + \frac{1}{10}u^{\frac{1}{10}})du$ (1)

(1) $= -\frac{2}{21}u^{\frac{21}{20}} + \frac{1}{11}u^{\frac{11}{10}} + C$

$= -\frac{2}{21}(1-\sqrt{t})^{21} + \frac{1}{11}(1-\sqrt{t})^{22} + C$ (1)

Evaluate $\int \frac{t^2+1}{(t-2)^2(t-3)}\,dt$.

SCORE: _____ / 6 PTS

$\frac{t^2+1}{(t-2)^2(t-3)} = \frac{A}{t-2} + \frac{B}{(t-2)^2} + \frac{C}{t-3}$

$t^2+1 = A(t-2)(t-3) + B(t-3) + C(t-2)^2$

$u=2: \quad 5 = -B \quad \Rightarrow \quad B = -5$

$u=3: \quad 10 = C$

COEF OF t^2: $\quad 1 = A + C \quad \Rightarrow \quad A = 1 - C = -9$

SANITY CHECK: $u=4: \quad \frac{17}{4(1)} \stackrel{?}{=} \frac{-9}{2} + \frac{-5}{4} + \frac{10}{1} = \frac{-18-5+40}{4} = \frac{17}{4}$ ✓

$\int \left(\frac{-9}{t-2} + \frac{-5}{(t-2)^2} + \frac{10}{t-3}\right) dt = -9\ln|t-2| + \frac{5}{t-2} + 10\ln|t-3| + C$

Evaluate $\int \frac{dt}{1-\cos t}$.

SCORE: _____ / 5 PTS

$$\int \frac{1}{1-\cos t}\frac{1+\cos t}{1+\cos t}\,dt = \int \frac{1+\cos t}{1-\cos^2 t}\,dt = \int \frac{1+\cos t}{\sin^2 t}\,dt = \int (\csc^2 t + \csc t \cot t)\,dt = -\cot t - \csc t + C$$

(2) (1½) (1½)