

Evaluate $\int \frac{dt}{1-e^t}$.

SCORE: ____ / 6 PTS

SOLUTION 1:

① $u = 1 - e^t \Rightarrow t = \ln(1 - u)$

$dt = \frac{1}{u-1} du$

② $\int \frac{1}{u(u-1)} du = \int (\frac{A}{u} + \frac{B}{u-1}) du$

③ $1 = A(u-1) + Bu$

$u = 1: 1 = B$

$u = 0: 1 = -A \Rightarrow A = -1$

SANITY CHECK: $u = 2: \frac{1}{2(1)} = \frac{-1}{2} + \frac{1}{1} = \frac{-1+2}{2} = \frac{1}{2} \checkmark$ ④

① $\int (-\frac{1}{u} + \frac{1}{u-1}) du = -\ln|u| + \ln|u-1| + C$

$= -\ln|1 - e^t| + \ln|-e^t| + C$ ⑤

$= t - \ln|1 - e^t| + C$ ⑥

SOLUTION 2:

$\int \frac{1}{1-e^t} \frac{e^{-t}}{e^{-t}} dt = \int \frac{e^{-t}}{e^{-t}-1} dt$ ②

$u = e^{-t} - 1$ ①

$du = -e^{-t} dt$

① $\int -\frac{1}{u} du = -\ln|u| + C$ ①

$= -\ln|e^{-t} - 1| + C$

①

Evaluate $\int \ln(t^2 - t + 2) dt$.

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$$u = \ln(t^2 - t + 2) \quad dv = dt$$

$$du = \frac{2t-1}{t^2-t+2} dt \quad v = t$$

$$\underbrace{t \ln(t^2 - t + 2)} - \underbrace{\int \frac{2t^2-t}{t^2-t+2} dt}$$

$$= t \ln(t^2 - t + 2) - \int \underbrace{\left(2 + \frac{t-4}{t^2-t+2}\right) dt}$$

$$= t \ln(t^2 - t + 2) - \int \left(2 + \underbrace{\frac{\frac{1}{2}(2t-1) - \frac{7}{2}}{t^2-t+2}}\right) dt$$

$$= t \ln(t^2 - t + 2) - 2t - \frac{1}{2} \ln(t^2 - t + 2) + \frac{7}{2} \int \underbrace{\frac{1}{(t-\frac{1}{2})^2 + \frac{7}{4}}} dt$$

$$= t \ln(t^2 - t + 2) - 2t - \frac{1}{2} \ln(t^2 - t + 2) + \frac{7}{2} \frac{2}{\sqrt{7}} \tan^{-1} \frac{t-\frac{1}{2}}{\frac{\sqrt{7}}{2}} + C$$

$$= \underbrace{t \ln(t^2 - t + 2)} - \underbrace{\frac{1}{2} \ln(t^2 - t + 2)} + \underbrace{\sqrt{7} \tan^{-1} \frac{2t-1}{\sqrt{7}}} - 2t + C$$

① EACH

Evaluate $\int (1 - \sqrt{t})^{20} dt$.

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SOLUTION 1:

$$\textcircled{1} \quad u = 1 - \sqrt{t} \Rightarrow t = (1 - u)^2$$
$$dt = -2(1 - u)du$$

$$\textcircled{1} \quad \int -2(1 - u)u^{20} du = \int (-2u^{20} + 2u^{21}) du$$

$$\textcircled{1} \quad = -\frac{2}{21}u^{21} + \frac{1}{11}u^{22} + C$$

$$= -\frac{2}{21}(1 - \sqrt{t})^{21} + \frac{1}{11}(1 - \sqrt{t})^{22} + C$$

SOLUTION 2:

$$\textcircled{1} \quad u = (1 - \sqrt{t})^{20} \Rightarrow t = (1 - u^{\frac{1}{20}})^2$$
$$dt = -\frac{1}{10}u^{-\frac{19}{20}}(1 - u^{\frac{1}{20}})du$$

$$\textcircled{1} \quad \int -\frac{1}{10}u^{\frac{1}{20}}(1 - u^{\frac{1}{20}})du = \int (-\frac{1}{10}u^{\frac{1}{20}} + \frac{1}{10}u^{\frac{11}{10}})du$$

$$= -\frac{2}{21}u^{\frac{21}{20}} + \frac{1}{11}u^{\frac{11}{10}} + C$$

$$= -\frac{2}{21}(1 - \sqrt{t})^{21} + \frac{1}{11}(1 - \sqrt{t})^{22} + C$$

OR

Evaluate $\int \frac{t^2 + 1}{(t-2)^2(t-3)} dt$.

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$$\frac{t^2 + 1}{(t-2)^2(t-3)} = \frac{A}{t-2} + \frac{B}{(t-2)^2} + \frac{C}{t-3}$$

$$\textcircled{1} \quad t^2 + 1 = A(t-2)(t-3) + B(t-3) + C(t-2)^2$$

$$u=2: \quad 5 = -B \Rightarrow B = -5$$

$$u=3: \quad 10 = C$$

$$\text{COEF OF } t^2: \quad 1 = A + C \Rightarrow A = 1 - C = -9$$

$$\text{SANITY CHECK: } u=4: \quad \overset{?}{\frac{17}{4(1)}} = \frac{-9}{2} + \frac{-5}{4} + \frac{10}{1} = \frac{-18-5+40}{4} = \frac{17}{4} \checkmark \textcircled{\frac{1}{2}}$$

$$\textcircled{1} \int \left(\frac{-9}{t-2} + \frac{-5}{(t-2)^2} + \frac{10}{t-3} \right) dt = -9 \ln|t-2| + \frac{5}{t-2} + 10 \ln|t-3| + C$$

$\textcircled{\frac{1}{2}} \quad \textcircled{1} \quad \textcircled{1} \quad \textcircled{\frac{1}{2}} \quad \textcircled{1} \quad \textcircled{\frac{1}{2}}$

Evaluate $\int \frac{dt}{1 - \cos t}$.

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$$\int \frac{1}{1 - \cos t} \frac{1 + \cos t}{1 + \cos t} dt = \int \frac{1 + \cos t}{1 - \cos^2 t} dt = \int \frac{1 + \cos t}{\sin^2 t} dt = \int (\csc^2 t + \csc t \cot t) dt = -\cot t - \csc t + C$$

Handwritten annotations:

- A red circle with the number 2 is above the first integral.
- A red circle with $1\frac{1}{2}$ is above the second integral.
- A red circle with $1\frac{1}{2}$ is above the third integral.
- Red brackets are drawn under the integrands of the second, third, and fourth integrals.